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Technical Notes

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Flame Temperature Distribution Measurement of Solid Propellants

Wenhua Zhao,* Shuguang Zhu,† Yan Li,‡
and Zhongyan Fang§
Tsinghua University,
100084 Beijing, People's Republic of China
and
Rongjie Yang¶ and Yuping Li¶
Beijing Institute of Technology,
100081 Beijing, People's Republic of China

Nomenclature

A_{ul} = transition probability
E_u = energy of the upper level
f_{lu} = oscillator strength
g_u, g_l = statistical weight
h = Planck constant
k = Boltzmann constant
$N(T)$ = number density of particles
T = temperature
$U(T)$ = partition function
λ = wavelength of the spectral line
ν = frequency of the spectral line

Subscripts

l = lower energy level
u = upper energy level

Superscripts

$'$ = spectral line N II: 701.398 nm
$''$ = spectral line N II: 701.473 nm

Introduction

It is important to measure the flame temperature for the combustion mechanism study of the solid propellants. Some methods, such as the absorption–emission spectroscopy,[1] the Fourier transform infrared absorption spectroscopy,[2] and the infrared fiber-optic diagnostic,[3] are used to diagnose the temperature. These methods exhibit many advantages over conventional methods such as thermocouples. But a few of these studies quantitatively measured the profiles of gas-phase temperatures.

In some cases thermocouples[4] are also used to get the temperature. But the low melting point, the heat loss from radiation and conduction, and the heat capacity limit the use of the thermocouples.

The spectroscopy method is used in this Note for the particular experimental condition. This method enables us to obtain simultaneously a large amount of information about the transient flame as well as stationary one. The emission spectrum method is used in the spectroscopy diagnostic for its simplicity and directness.

In this Note the flame temperature of the steady burning stage is measured. A special relative intensity method using the combination of two closely spaced spectral lines with a third line more separated in the spectrum, which has not been reported before, is employed for the temperature distribution measurement of the solid-propellant combustion flame, and satisfactory results at different pressures are obtained.

Overview

In fact, the spectrum of the solid-propellant flame is very complex.[5] Based on the spectrum analysis of the solid-propellant combustion flame, the relative intensity method is developed in this Note, in which three nitrogen ion spectral lines are used; two of them are very close and are difficult to distinguish.

The characteristic of the emission spectrum of the solid-propellants flame is that there are a series of spectral lines and a weak continuous spectrum from the wavelength 500 to 800 nm.[5]

If the optically thin flame in the local thermodynamic equilibrium state is considered, the emission coefficient ε_ν corresponding to a line frequency ν by atoms or ions, which undergo a transition from the upper energy level u to the lower energy level l, is given by Eq. (1)[6]:

$$\varepsilon_\nu = (h\nu/4\pi)A_{\mathrm{ul}}g_u[N(T)/U(T)]\exp(-E_u/kT) \tag{1}$$

where the emission coefficient ε_ν is the power emitted per unit solid angle and unit volume by a spectral line of frequency ν, A_{ul} is transition probability for emission. The dimensionless number f_{lu} is related to A_{ul} by the formula[7]

$$A_{\mathrm{ul}} = 7.407 \times 10^{-22}\nu_{\mathrm{ul}}^2(g_l/g_u)f_{\mathrm{lu}} \tag{2}$$

So, Eq. (1) can be written as

$$\varepsilon_\nu = 5.894 \times 10^{-23}h\nu^3 g_l f_{\mathrm{lu}}[N(T)/U(T)]\exp(-E_u/kT) \tag{3}$$

The measured axisymmetrical combustion flame in this paper is a nonuniform body, and Abel's transformation is used to get the emission coefficient ε_ν of the axisymmetrical flame from the spectral line intensity.

If the intensity of two spectral lines is measured simultaneously, the temperature could be calculated through the relative intensity method. The relative intensity of the same particle is expressed as

$$\frac{\varepsilon_\nu}{\varepsilon'_\nu} = \frac{g_l f_{\mathrm{lu}}\nu^3}{g'_l f'_{\mathrm{lu}}\nu'^3}\exp\left(\frac{E'_u - E_u}{T}\right) \tag{4}$$

If the wavelengths of two spectral lines are very close, for example, the difference of the wavelength is only 0.1 nm, they are

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*Professor, Thermophysics Institute, Department of Engineering Mechanics; zhaowh@mail.tsinghua.edu.cn.

†Ph.D. Student, Thermophysics Institute, Department of Engineering Mechanics; zhushuguang98@mail.tsinghua.edu.cn.

‡Adjunct Professor, Department of Precision Instruments and Mechanology.

§Professor, Department of Precision Instruments and Mechanology.

¶Professor, Institute of Materials Science and Engineering.

difficult to distinguish by the photovoltaic conversion unit, which limited the use of the spectral line intensity method. To deal with this problem, the relative intensity method using two spectral lines are adjusted in this Note, in which the ratio of the total intensity for the serried spectral lines to another spectral line intensity is used to calculate the temperature.

The wavelengths of three spectral lines used in this Note are 624.241, 701.398, and 701.473 nm (Ref. 8). Although the wavelengths of the last two spectral lines are close, they can be used because all of three spectral lines are from the nitrogen ion.

The relationship between the emission coefficient $\varepsilon_\nu(\varepsilon'_\nu, \varepsilon''_\nu)$ and the temperature T of the flame for three chosen spectral lines could be expressed as

$$\varepsilon_\nu = 5.894 \times 10^{-23} h\nu^3 g_l f_{\mathrm{lu}}[N(T)/U(T)]\exp(-E_u/kT) \tag{5}$$

$$\varepsilon'_\nu = 5.894 \times 10^{-23} h\nu'^3 g'_l f^{;}_{\mathrm{lu}}[N(T)/U(T)]\exp(-E'_u/kT) \tag{6}$$

$$\varepsilon''_\nu = 5.894 \times 10^{-23} h\nu''^3 g''_l f''_{\mathrm{lu}}[N(T)/U(T)]\exp(-E''_u/kT) \tag{7}$$

The close spectral lines have the same energy of high level in our case, and so the sum of Eqs. (6) and (7) can be simplified as

$$\varepsilon'_\nu + \varepsilon''_\nu = 5.894 \times 10^{-23} \frac{hN(T)}{U(T)} \times \left(g'_l f'_{\mathrm{lu}} \nu'^3 + g''_l f''_{\mathrm{lu}} \nu''^3\right) \exp\left(-\frac{E'_u}{kT}\right) \tag{8}$$

The equation for the improved relative intensity method is written as

$$\frac{\varepsilon_\nu}{\varepsilon'_\nu + \varepsilon''_\nu} = \frac{g_l f_{\mathrm{lu}} \nu^3}{\left(g'_l f'_{\mathrm{lu}} + g''_l f''_{\mathrm{lu}}\right)\nu'^3} \exp\left(\frac{E'_u - E_u}{kT}\right) \tag{9}$$

where

$$T = E'_u - E_u \Big/ k \ln \frac{\varepsilon_\nu \left(g'_l f'_{\mathrm{lu}} + g''_l f''_{\mathrm{lu}}\right)\nu'^3}{\left(\varepsilon'_\nu + \varepsilon''_\nu\right) g_l f_{\mathrm{lu}} \nu^3} \tag{10}$$

Abel's transformation is used to get the emission coefficient distribution in the radial direction from the measured spectral line intensity as many works have done.[6] The Abel inversion to "noisy" data can lead to very large uncertainty. To get the better data and reduce this uncertainty, polynomial fit is made to the spectral line intensity distribution in the cross section before the Abel inversion. Although the polynomial is made, the Abel inversion can lead to error, and then the temperature distribution in the radial direction can be calculated by Eq. (10). The temperature distribution in the axes can be easily acquired after the temperature distributions in the radial direction of all sections are calculated.

Experiment

The vessel for the measurement is a cylinder container that was designed to work at high pressures up to 20 MPa.

The spectrum diagnostic system includes the optical scanning system by which the temperature distribution can be measured, the high-sensitivity photovoltaic conversion unit, the high-performance amplifying unit through which transient feeble signals can be detected, and the data acquisition and processing unit from which the measured results will be given. The sketch of the spectroscopic system is shown in Fig. 1. The target investigated in the vessel 1 is located in front of the scanning system 2, by which the image of the target would be formed at the entrance slit of the spectrometer 4. Optical signals are transformed into the electrical signals by the photodiodes 5 at the exit window of the spectrometer, and then the electrical signals are amplified by operational amplifiers 6. The amplified signals are consequently input to the A/D converter 8 and then the PC system 9.

The scanning system is made up of a 30-facet mirror drum, the motor, and the rotation speed controller. The effect of the translational movement of the mirrors on the formation of the image can be neglected. The optical arrangement, which consists of lenses and the mirror drum, ensures that the output image of the scanning unit will not be deformed when the measured is randomly moving in a known space.

The intensity $I(\nu)$ of one given frequency spectral line is converted to voltage V through the scanning unit, spectrometer, photodiode, and amplifier. It can be written as $V = I(\nu) \times G(\nu)$, where $G(\nu)$ is a general factor including the instrument factors. It is the function of the frequency and can be considered as a constant for the given frequency. It is calibrated by the standard tungsten strip lamp before the measurement. The $I(\nu)$ can be calculated easily via $G(\nu)$ in the experiment from the voltage sampled by the A/D system. The emission coefficient ε_ν is then known by Abel's transformation.[7]

The samples used in the experiment are the double-based propellant SQ-2, which are 25 mm long and 5 mm in diameter. The weight of the samples is about 834 mg. To prevent the explosive combustion and ensure the axis symmetry of the flame, the samples are coated. The propellants are ignited by a resistance line, which is heated by the current. The vessel is filled with nitrogen gas, which forms the inert environment.

The duration of the whole burning process of SQ-2 propellant samples at different pressure are all larger than 3 s. To ensure the steady-state burning progress, the measurement is made 1 s after ignition. The whole measurement can be completed within 1 ms,

Table 1 Comparison of three methods for the SQ-2 propellant

Work pressure, MPa	Spectroscopy method, K	Thermocouple method, K	Equilibrium calculation, K
3	2038	1943	2330.9
5	2148	2048	2334.7
7	2227	2132	2336.8

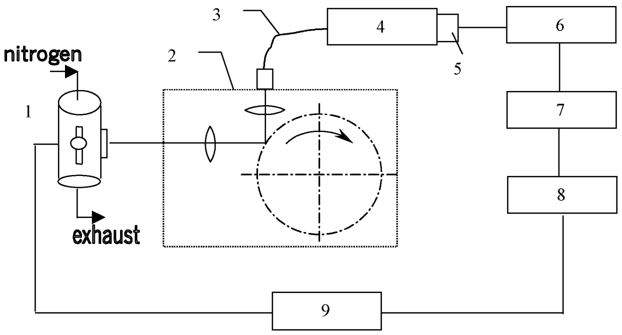

Fig. 1 Sketch of measurement system: 1, vessel; 2, optical scanning system; 3, optical fiber; 4, grating spectrograph; 5, photovoltaic conversion unit; 6, amplifer; 7, A/D converter; 8, computer; and 9, trigger.

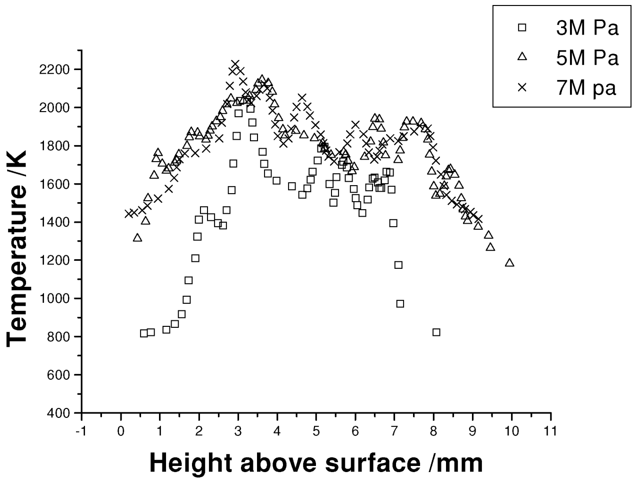

Fig. 2 Temperature distribution for SQ-2 propellant by spectroscopy method at different pressures.

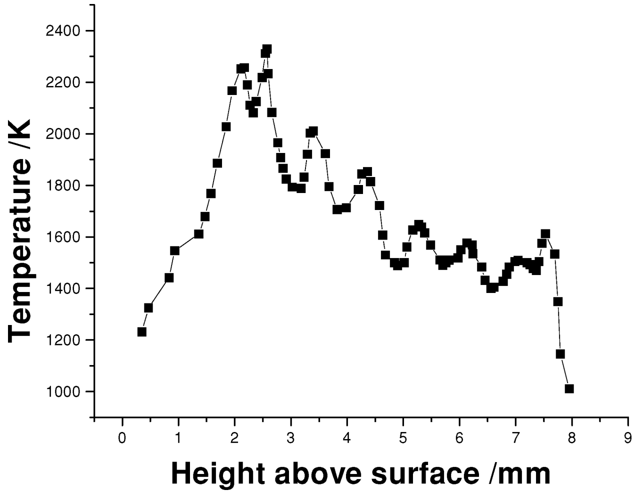

Fig. 3 Temperature distribution for NEPE propellant by spectroscopy method at 3-MPa pressure.

and the maximum time resolution for the temperature measurement of one point is about 10 μs, which is much smaller than the burning duration of SQ-2 propellant samples.

Conclusions

All results in this paper are given based on the following assumptions: 1) combustion of the flame is steady; 2) the flame is optically thin for its small size; and 3) the flame is in local thermal equilibrium state.

The temperature distributions in the axes for the SQ-2 propellant at different pressures are shown in Fig. 2. There are three areas, the climbing-temperature area, the high-temperature area and the fall-temperature area, in the temperature distribution. The temperature in the high-temperature area is unstable because of the turbulence of the flame and the unsteady of combustion. And also, this instability can be found in the line intensity data without Abel's inversion.

The maximum flame temperatures are compared with the results from thermocouple method and equilibrium calculation (shown in Table 1). The data of thermocouple method and equilibrium calculation are provided by the Beijing Institute of Technology.

The data in Table 1 show that the maximum flame temperature increases along with the rising of the pressure. The results from the spectroscopy method are about 100 K higher than those from the thermocouple method and much nearer to the temperature from equilibrium calculation with the rising of the pressure. As a reference, Dong Yang[1] has reported that the maximum temperatures of SQ-2 propellant exhaust plumes are 2234 and 2202 K for 240 and 500 mm apart from the motor nozzle when the work pressure of the chamber is about 13.0 MPa. The results from the spectroscopic diagnostic system are reasonable in comparison with the preceding results from other methods. The relative standard deviations of maximum temperature of the SQ-2 propellant's flame at different pressures are less than 5% for all measurements.

The temperature distribution of the solid-propellant nitrate-ester-plasticized polyethane (NEPE) is also measured by the spectroscopic diagnostic system. The result is not satisfactory because there are metal particles in the NEPE propellant, which will absorb and scatter the radiation, and emit their own spectrum. Figure 3 shows one of the measurement results. In fact, the temperature measurements for the NEPE propellant at different pressures are made for many times. The relative standard deviations of the maximum flame temperature are about 15% in our measurements, which are a little bit larger. This will be studied in the later work.

Acknowledgments

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References

[1] Yang, D., Xu, H. Q., Wang, J. D., and Zhao, B. C, "Temperature Measurement of Solid Rocket Motor Exhaust Plume by Absorption-Emission Spectroscopy," *Spectroscopy Letters*, Vol. 34, No. 2, 2001, pp. 109–116.

[2] Mallery, C. F., and Thynell, S. T., "Species and Temperature Profiles of Propellant Flames Obtained from FTIR Absorption Spectroscopy," *Combustion Science and Technology*, Vol. 122, No. 1–6, 1997, pp. 113–129.

[3] Wormhoudt, J., Kebabian, P. L., and Kolb, C. E., "Infrared Fiber-Optic Diagnostic Observations of Solid Propellant Combustion," *Combustion and Flame*, Vol. 108, No. 1–2, 1997, pp. 43–60.

[4] Zenin, A. A., and Finyarkov, S. V., "Study of Solid-Propellant Ignition by a Hot Gas-Stream," *Combustion Explosion and Shock Waves*, Vol. 29, No. 3, 1993, pp. 270–275.

[5] Zhao, W., Zhu, S., Tian, K., and Liu, D., "Quick Spectroscopy Diagnostics for the Flame Temperature," *Combustion of Energetic Materials*, edited by K. K. Kuo and L. T. Deluco, Begell House, New York, 2002, pp. 753–761.

[6] Venugopalan, M. (ed.), *Reactions Under Plasma Conditions*, Vol. 1, Wiley, New York, 1971, p. 389.

[7] Venugopalan, M. (ed.), *Reactions Under Plasma Conditions*, Vol. 1, Wiley, New York, 1971, p. 105.

[8] Kasabov, G. A., and Eliseev, V. V., *Spectrum Table for Low Temperature Plasma*, Moscow Atomic Energy Press, Moscow, 1973, pp. 160, 161.

Thermally Asymmetric Annular Rectangular Fin Optimization

H. S. Kang*
Kangwon National University,
Chunchon 200-701, Republic of Korea
and
D. C. Look Jr.†
University of Missouri–Rolla, Rolla, Missouri 65409-0050

Nomenclature

Bi = Biot number, hl'/k
h_j = heat transfer coefficient of surface j, W/m^2 K
k = thermal conductivity of fin material, W/m K
l' = one-half fin height at the base, m; dimensionless form, $l = l'/r_i'$
M_j = convection characteristic number of surface j, $h_j r_i'/k (= B_{j1}/l)$
q = heat transfer from the fin, W; dimensionless form, $Q = q/k\varphi_f(2\pi r_i')$
r' = radius, m; dimensionless form, $r = r'/r_i'$
r_i' = pipe inside radius, m
r_j' = j radius, m; dimensionless form, $r_j = r_j'/r_i'$

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*Associate Professor, Department of Mechanical Engineering; hkang1960@yahoo.com.

†Professor Emeritus, Department of Mechanical and Aerospace Engineering and Engineering Mechanics; look@umr.edu. Associate Fellow AIAA.